PROBLEM SET 6

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

1) **(5 pts)** **TextProblem 6.5**

2) **(10 pts)** A system has three non-degenerate energy levels with energies 0, ε, and 2ε.

(*hint:* Set up similar to problem 6 of HW set 5. How many ways (microstates) can the particles populate the 3 energy levels such that the total energy is 2ε, then the entropy can be easily calculated)

a) Calculate the entropy of the system if the three levels are populated by two distinguishable particles such that the total energy is U=2ε.

b) Calculate the entropy of the system if the three levels are populated by three distinguishable particles such that the total energy is U=2ε.

3) **(5 pts)** **TextProblem 6.12**
(*hint:* Think of ratio of probability between 1st excited state and ground state, and note that 1st excited state has 3 degenerate states with the same energy. Also, note that the energy given is really a difference between 1st and ground)

4) **(5 pts)** **TextProblem 6.13**
(*hint:* Think of ratio of probabilities as the ratio of $P_{\text{neutron}}/P_{\text{proton}}$, you are also told how to calculate the total energy difference. Also keep in mind that $P_{\text{neutron}} + P_{\text{neutron}} = 1$)

5) **(25 pts)** Consider a system with two non-degenerate energy levels with energies $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon$. Suppose that the system contains N distinguishable particles at temperature T.

a) Determine the partition function of the system and the occupancies N_1 and N_2 of the two levels.

b) Find the average energy per particle given by $< u > = \mathrm{U/N}$, where U is the total internal energy of the system and N the total number of particles.

c) Show that at very small temperatures, $< u > \approx \epsilon e^{-\alpha}$, where $\alpha = \dfrac{\epsilon}{\mathrm{k_B T}}$, and that as the temperature becomes very large, $< u > \rightarrow \dfrac{1}{2}\epsilon$

d) Show that the volume heat capacity per particle is given by $\mathrm{C/N} = \mathrm{k_B}\alpha^2 e^{-\alpha}/(1 + e^{-\alpha})^2$

6) **(20 pts) Text Problem 6.31** – to perform the required integrals, make a change of variable and then carry out an integration by parts.

7) **(5 pts) Text Problem 6.33** – evaluate the requested quantities at $\mathrm{T} = 20^\circ$ C. (*Hint:* Careful with the units, convert to K first)

8) **(5 pts)** Calculate the average energy (in eV) and rms velocity of an electron at the the temperatures $\mathrm{T_1} = 10^3$ K and $\mathrm{T_2} = 10^5$ K

9) **(20 pts)** Suppose that instead of the Maxwell-Boltzmann distribution, the distribution of molecular speeds in a gas was given by the expression $\mathrm{N(v)} = \mathrm{Av}e^{-\mathrm{v/v_o}}$, where A and $\mathrm{v_o}$ are constants.

a) Determine the constant A so that the total number of molecules in the gas is N.

b) In terms of $\mathrm{v_o}$, find the average speed, the rms speed, and the most likely speed of the molecules in the gas.