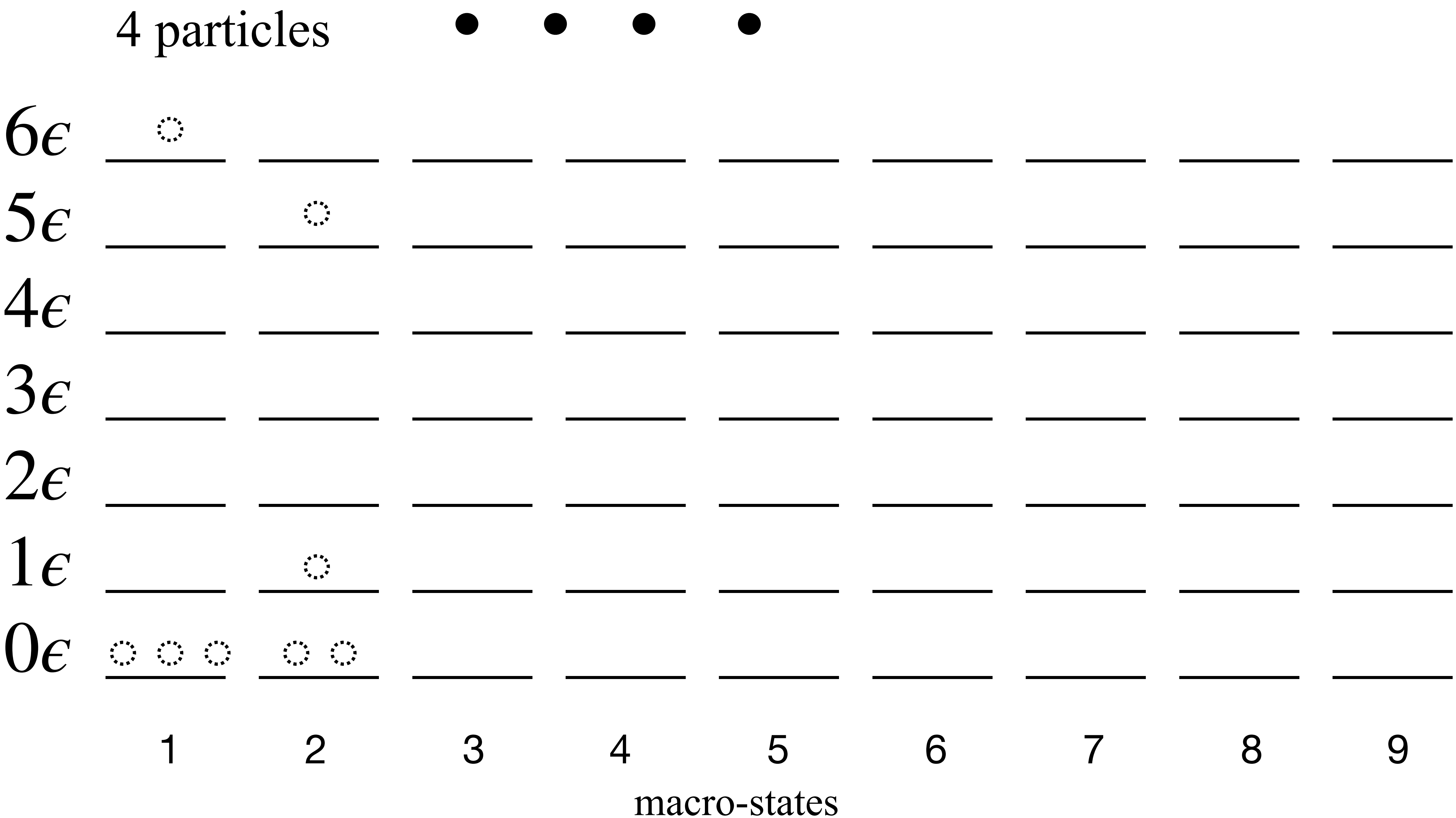
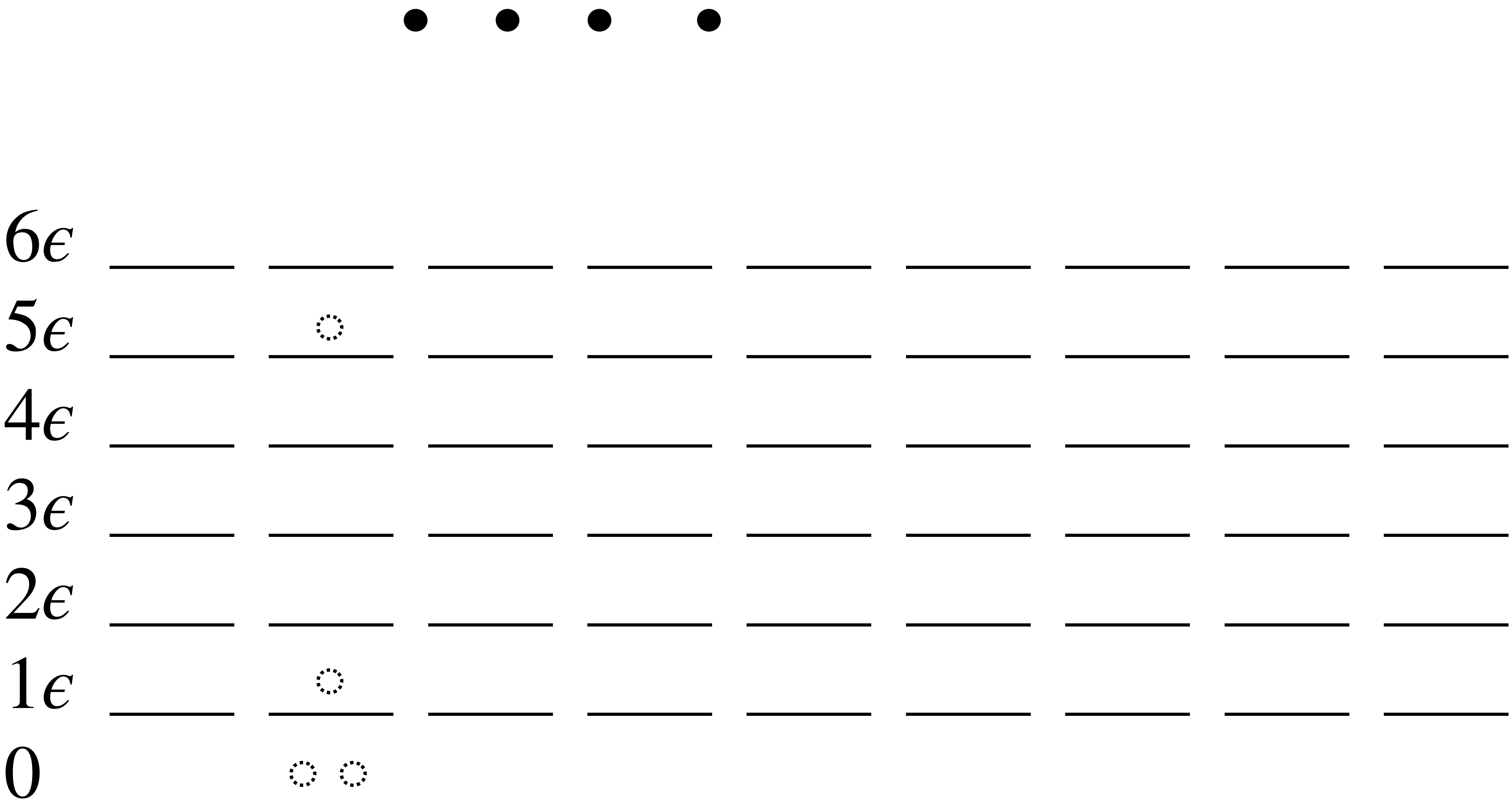
6) **(20 pts)** Consider a system with 7 energy levels with energies 0, ε, 2ε, 3ε, 4ε, 5ε, and 6ε, for some energy ε. Suppose it is desired to fill these energy levels with four particles such that the total energy of the system of four particles is 6ε. For example, we could put three particles in the level with zero energy and one particle in the level with energy 6ε.

a) Define macrostates of the system as states with different numbers of particles in each level. Then for the system described above with four particles and total energy 6ε, there are nine possible macrostates. Make a diagram of these macrostates, which shows for each macrostate, how many particles occupy each of the seven energy levels.

HW Set 5 Problem 6
Part A



b) Now suppose the particles are *distinguishable* so that we can determine which particles are in which state. Then for each macrostate, there exist several microstates corresponding to different choices for the particles in each level. For example, for a macrostate which has three particles in level 0 and one particle in level 6 (for a total energy of $3 \times 0\varepsilon + 1 \times 6\varepsilon = 6\varepsilon$), there are 4 choices for the first particle, 3 choices for the second particle, and 2 choices for the third particle in level 0, and only 1 choice for the last particle in level 6. But the order of choosing first 3 particles doesn't make any difference, so we have to divide by 3 x 2 =6, to account for the fact that the first particle can be placed in any of the 3 slots available in 0ε, and the second particle can be placed in any of the 2 slots left. Thus, for this macrostate, there are four different microstates and the multiplicity is Ω=4 for that macrostate. Determine the multiplicity of each macrostate according to this definition and then show that the sum of all nine multiplicities is 84. This is the total number of microstates in the system.



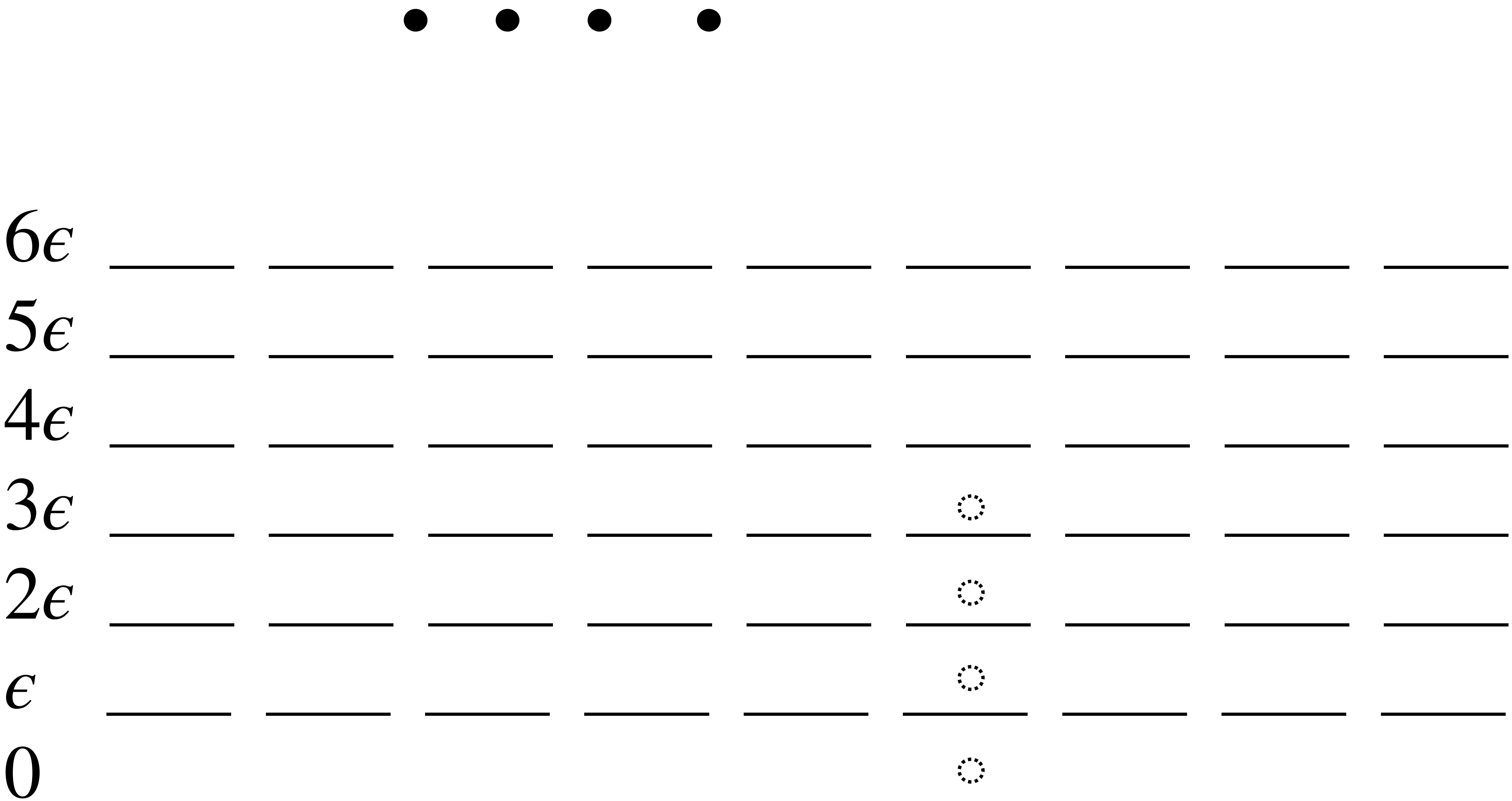
(4/2) * (3/1) * (2) * 1 = 12

Any of the 4 particles can be placed in any of the 2 slots at $\epsilon = 0$, in any order, hence (4/2)

Any of the 3 particles left can be placed in a single slot in $\epsilon = 0$, hence multiply by (3/1)

Any of the two particle left can be placed in the single slot $\epsilon = 1$, hence multiply by (2/1)

The only particle left can can be placed in the single slot $\epsilon = 5$



4*3*2*1 = 24

Any of the 4 particles can be placed in a single slot at $\epsilon = 0$, hence (4)
Any of the 3 particles left can be placed in a single slot in $\epsilon = 1$, hence multiply by (3)
Any of the 2 particles left can be placed in the single slot $\epsilon = 2$, hence multiply by (2)
The only particle left can can be placed in the single slot $\epsilon = 3$, hence multiply by 1