PROBLEM SET 4

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

1) **(20 pts)** A closed cylinder of total volume V is divided into three sections. The first section contains 1000 moles of helium, the second section 2000 moles of neon, and the third section 3000 moles of argon (all monatomic gases). The volumes of the three sections are chosen so that the pressures in each section are the same and equal to 2.0 atm. The temperatures in the three sections are also the same and equal to 300 K.

a) Express the volume of each section as a fraction of V.

b) The partitions dividing the sections are now removed allowing all three gases to occupy the full volume V. Calculate the resulting change in the entropy of the system. Keep in mind that the temperature stays constant during the expansion of the three gases.

c) Determine the resulting change in the Helmholtz free energy of the system. *Hint*: recall that the work done by an expanding gas is equal to the negative of its free energy change.

2) **(20 pts)** The Gibbs free energy of a non-ideal gas is given by the equation
$$G = n[RT\ \ln(P/P_0) - f(T)P]$$

where n is the number of moles of the gas, P_0 is a constant, and f(T) is some function of the temperature.

a) Using the relation between V and G, show that $PV = n[RT - f(T)P]$

b) Using the relation between S and G, find the entropy of the gas.

c) Determine the Helmholtz free energy (the function F) of the gas.

3) **(20 pts)** Recall that the equation of state for a van der Waals gas is given by

$$\left(P + \frac{aN^2}{V^2}\right)(V - bN) = NkT$$

where a and b are constants and N is the number of molecules of the gas.

a) Solve for the pressure in terms of V and T.

b) Suppose that N molecules of a van der Waals gas expand *isothermally* from volume V_1 to volume V_2. Calculate the change in the Helmholtz free energy.

c) By integrating the expression connecting P and F, derive an expression for F in terms of T and V. Note that the integration over V yields an integration constant that is a function of T. Now differentiate the result to find the entropy S as a function of V and T.

d) Using the results of the previous parts, show that the change in the internal energy of the gas during the isothermal expansion is given by
$\Delta U = aN^2(1/V_1 - 1/V_2)$

4) **(10 pts) TextProblem5.5**

5) **(5 pts) Text Problem 5.10** – assume here that neither the entropy nor the volume per mole of liquid water depend on the temperature within the range considered.

6) **(5 pts) Text Problem 5.12**

7) **(10 pts) Text Problem 5.32 –** in part b, use the value L=334 kJ/kg for the latent heat of fusion of water. In part d, use the value m=60 kg for the mass of the skater and assume that only one skate is in contact with the ice and that the part of the blade in contact with the ice is a rectangle with dimensions 30 cm x 0.5 cm.

8) **(10 pts) Text problem 5.35 –** note that the quantity L is here is the latent heat per mole rather than per kg, so in the Clausius-Clapeyron equation, you should replace m by n.